Parameter estimation of inspiralling compact binaries:

exhaustive comparison between theory and simulations: implications for searches in ground based detectors

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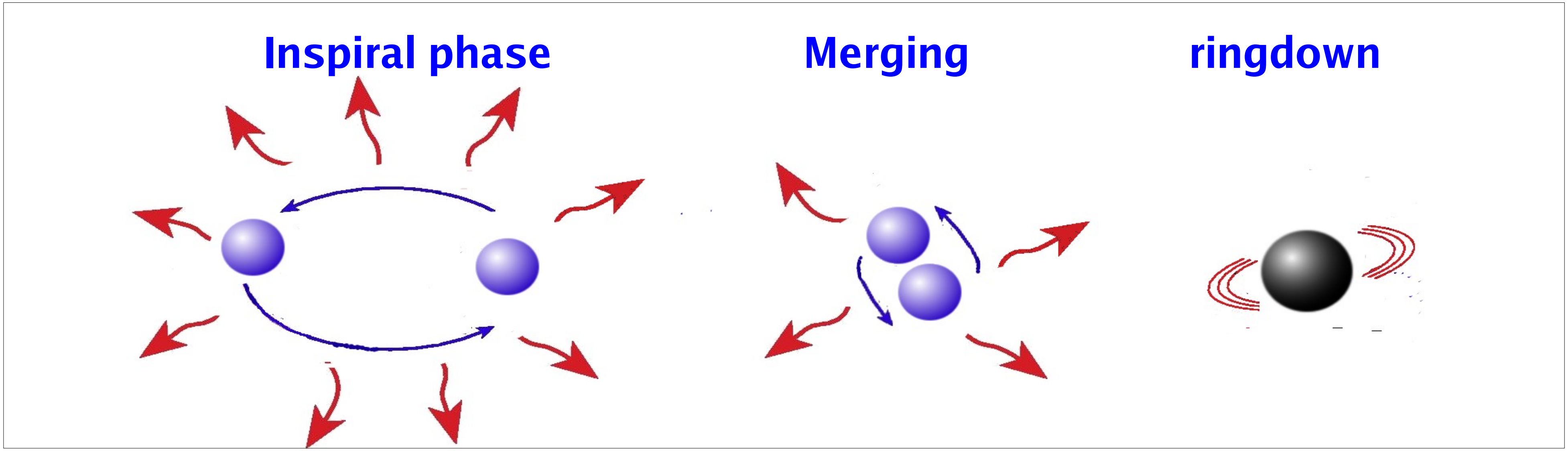
Overview and Motivation

Overview: target waveforms and detectors

We consider *non-spinning compact binaries* (restricted amplitude), with phase order at 2 and 3.5PN.

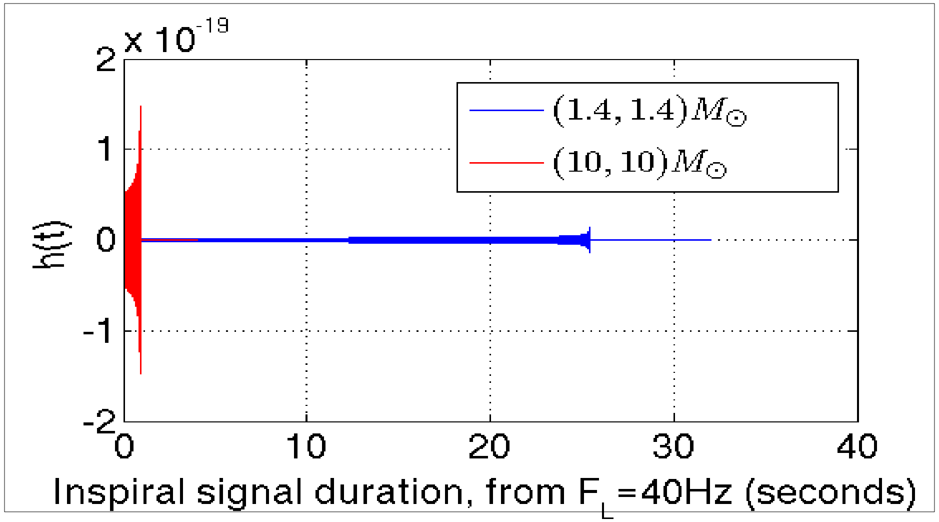
In particular, we consider binary black hole (BBH), binary neutron stars (BNS) and black-hole – neutron-star binary (BHNS).

We consider the inspiral phase only, and initial *LIGO design sensitivity curve.*



Overview: target waveforms and detectors

BBH, BNS and BH-NS are quite different in nature BUT signals in time domain are generated using the following (simplified) expression, which depends only on a pair of mass parameters (e.g, m1,m2):



$$h(t) = \frac{1Mpc}{D_{\text{eff}}}\left[h_c(t)\cos\Phi + h_s(t)\sin\Phi\right]$$

We can use the stationary phase approximation to obtain an analytical expression in the frequency domain. The frequency counterpart can be used to derive the covariance matrix and therefore analytical expression of the errors in parameters estimation (e.g., Arun et al, PRD 71, 084008, 2005).

Overview: detection

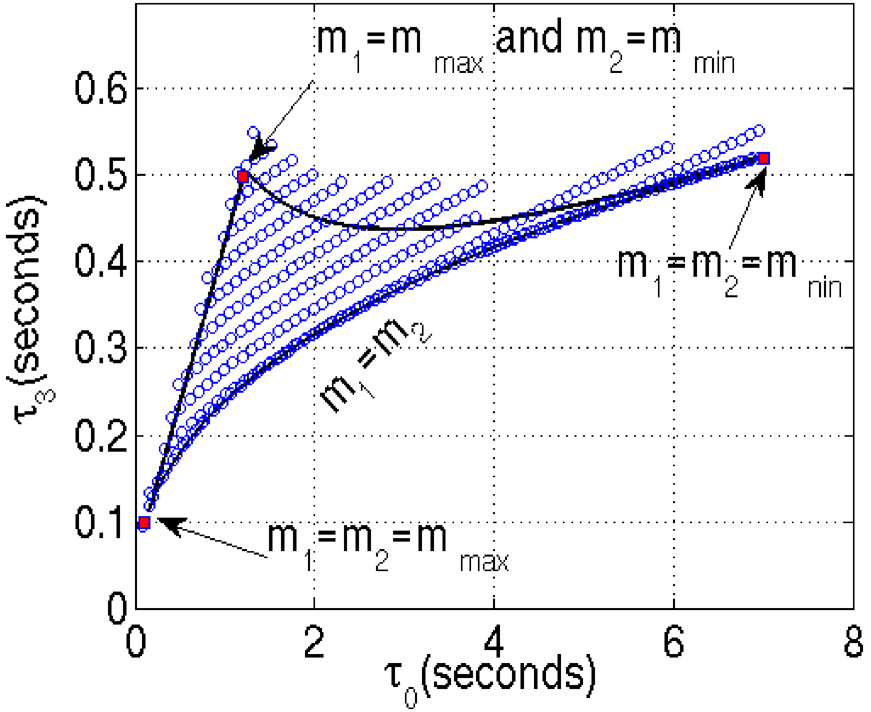
For parameter estimation, we usually use chirp mass and eta parameters :

$$\mathcal{M} = M\eta^{3/5} \qquad \eta = \frac{m_1 m_2}{M^2}$$

For detection (template placement), we usually work with tau parameters :

$$\tau_0 = \frac{A_0}{\eta} M^{-5/3} \qquad \tau_3 = \frac{A_3}{\eta} M^{-2/3}$$

!!! Eta greater than 0.25 leads to unphysical mass components. Usually, corresponding templates are not used because m1, m2 would be non-real.



Overview: motivation

For both detection and parameter estimation, we compute matches between (signal+noise) and templates, using this known expression:

$$< h_1 | h_2 >= \int df \frac{\tilde{h}_1(f)\, \tilde{h}_2^*(f)}{S(f)}$$

Where h(f) is given by the stationary phase approximation if possible!

Motivation

Because the covariance matrix is valid in the strong signal approximation case only, we want to perform Monte Carlo simulations to estimate the discrepancies at low SNR.

Motivation

Especially important because first detections might be at low SNR only. Moreover, coincidences between detectors are done assuming that the errors given by the covariance matrix are correct. What is really happening at low SNR ?

Simulations and results

Protocol

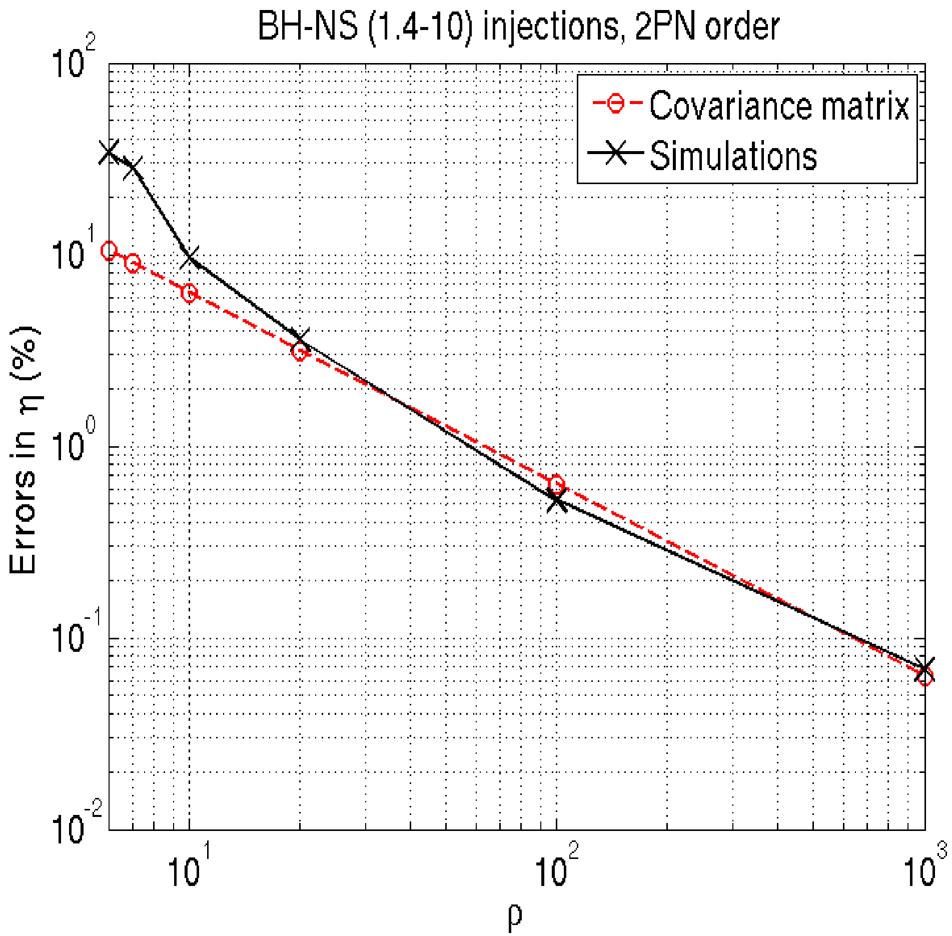
- Generate gaussian noise with initial LIGO power spectrum density
- inject a signal with fixed masses m1 and m2 (e.g., 10,10) and a fixed SNR.
- filter the noise+signal with a set of templates.
- keep the parameter of the template that gives the largest SNR

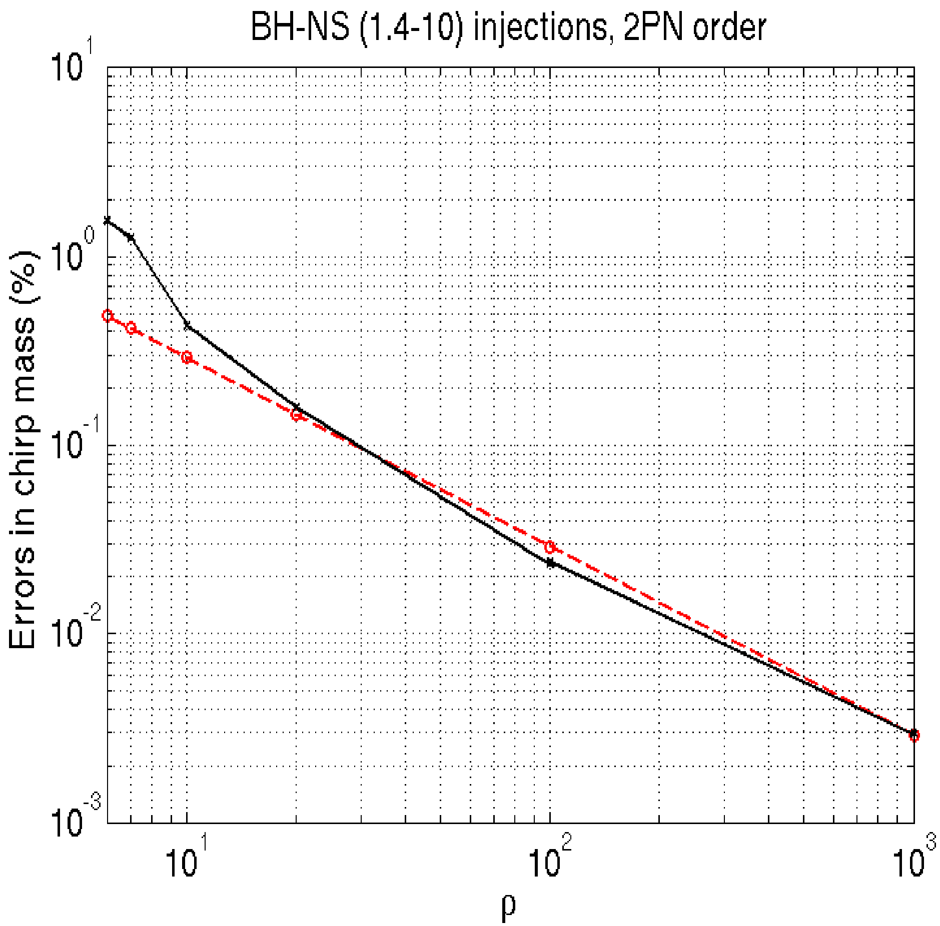
$$\text{match} = \max_{t_0,\phi_0,\mathcal{M},\eta,\ldots} \frac{\langle h | h_{\text{true}} \rangle}{\sqrt{\langle h | h \rangle \langle h_{\text{true}} | h_{\text{true}} \rangle}}$$

- Repeat 4 first steps as much as needed (here, 10 000 times)
- compare the errors on parameters with expected errors from covariance studies.

First simulations : BH-NS injections

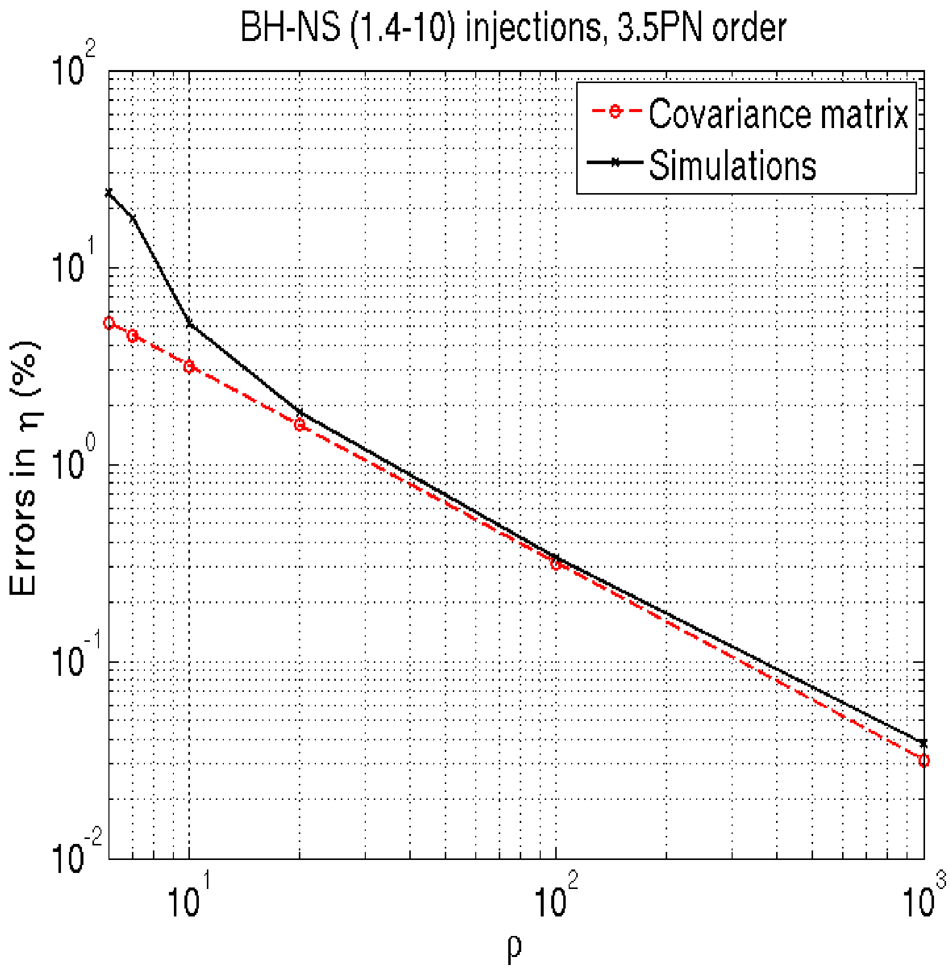
BH-NS (10,1.4) injections with phase at 2PN order. Simulations agrees with covariance results for SNR > 20 up to 1000 at least.

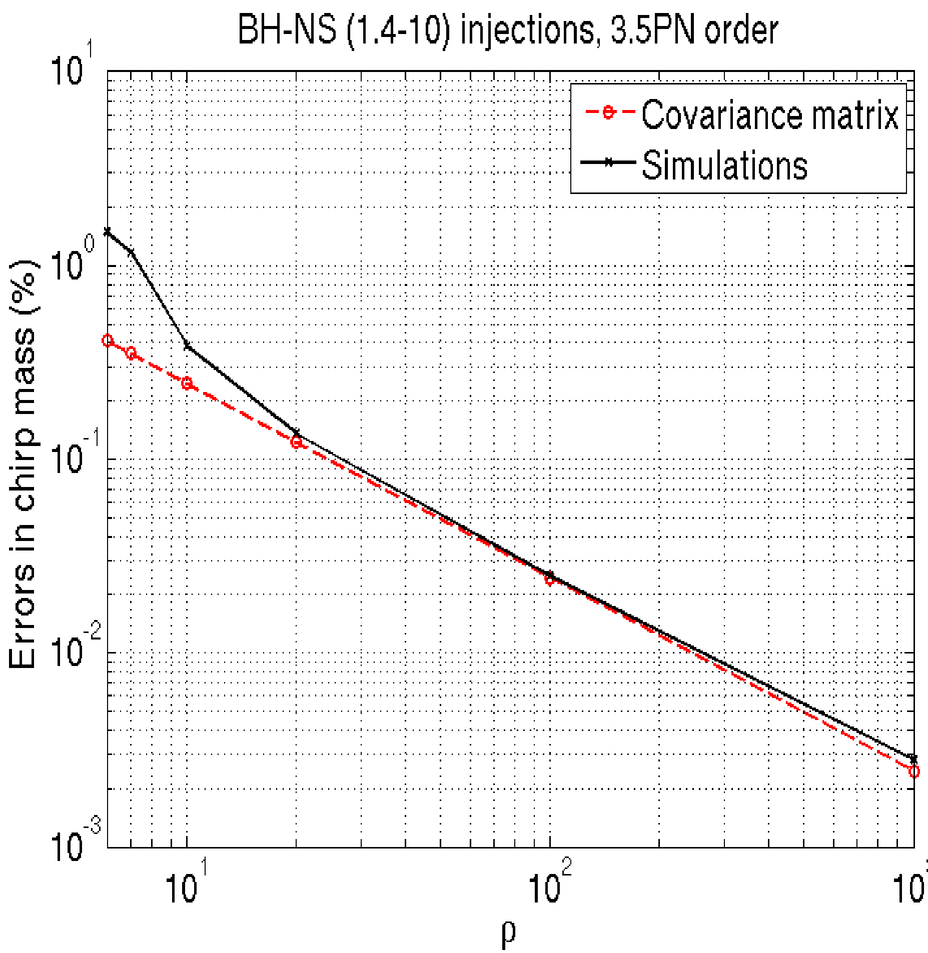




First simulations : BH-NS injections

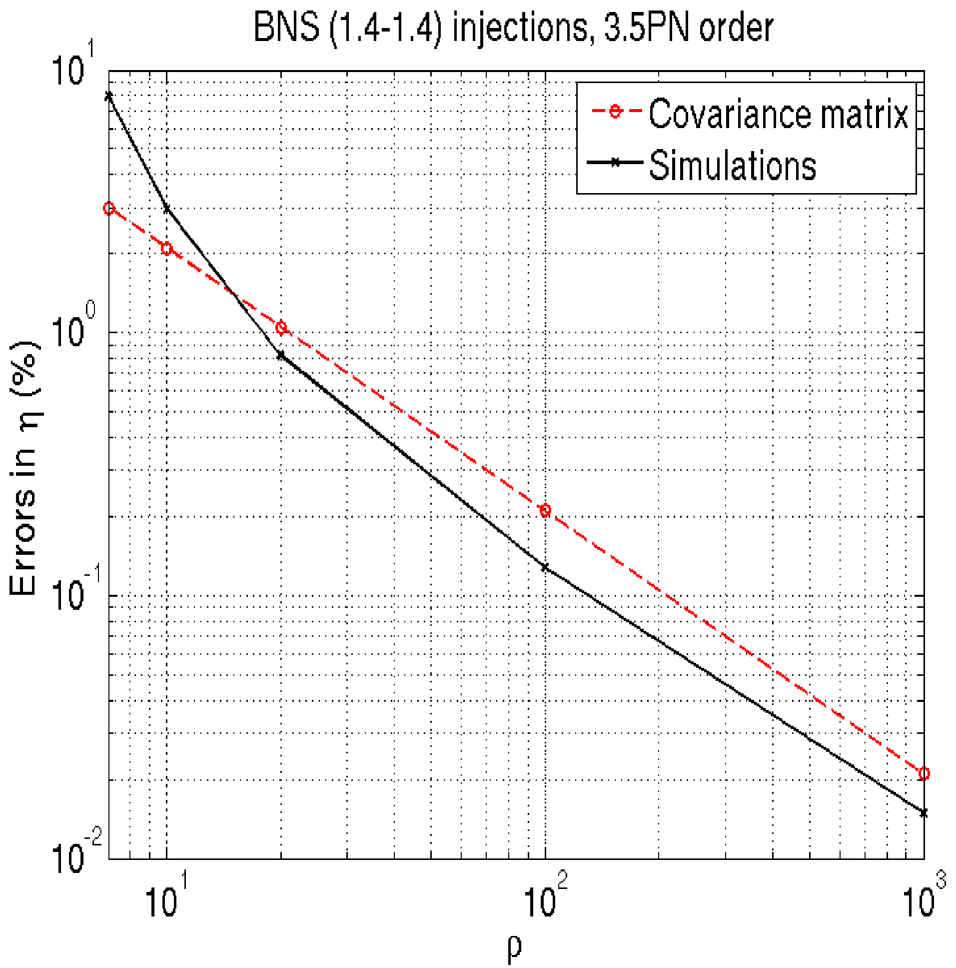
BH-NS (10,1.4) injections with phase at 3.5PN order. Simulations agrees with covariance results for SNR > 20 up to 1000 at least.

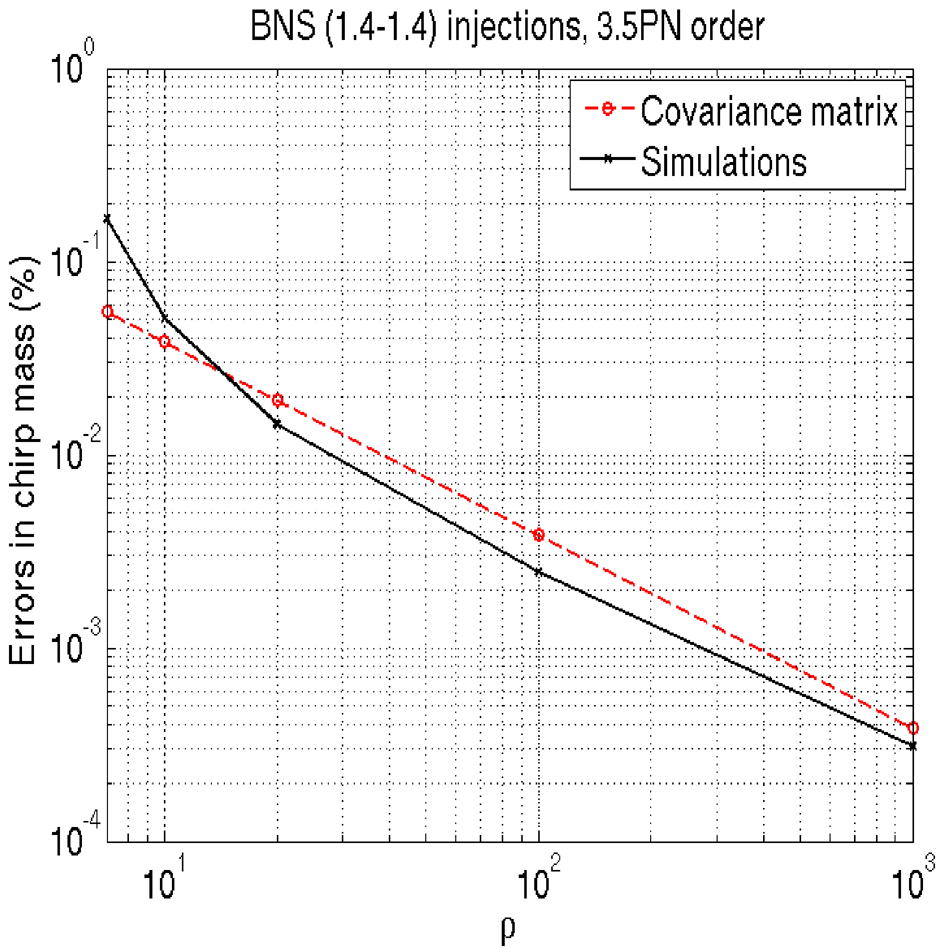




BNS simulations

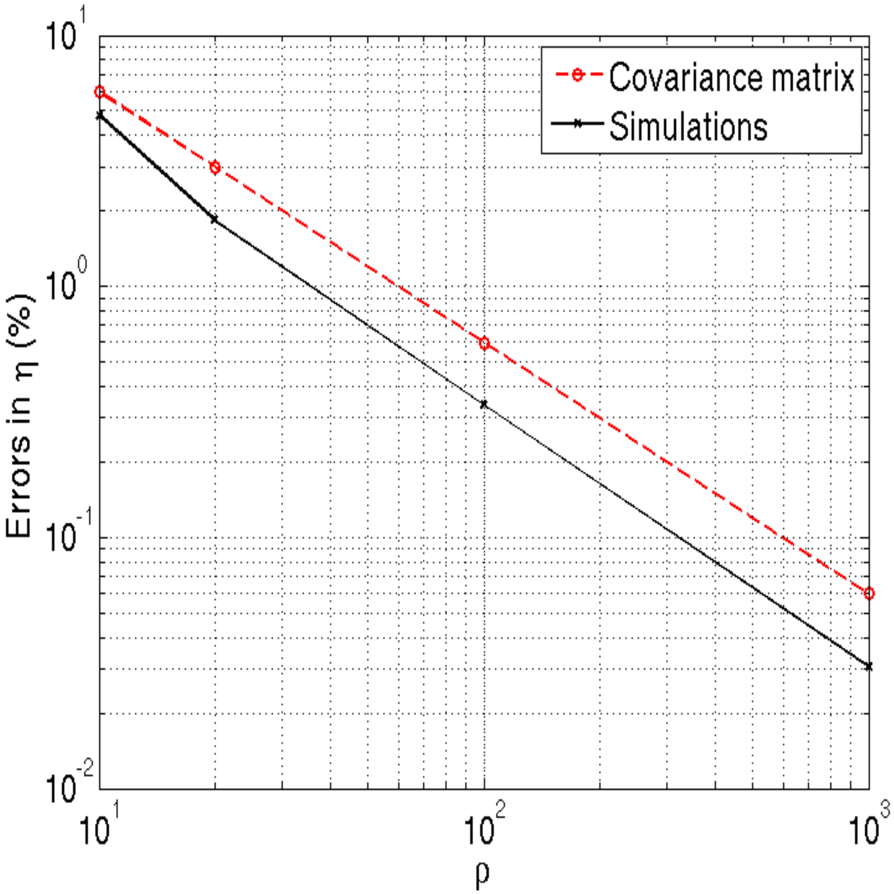
BNS (1.4,1.4) injections with phase at 3.5PN order. Simulations DO NOT agree with covariance results. We have a better accuracy with our simulations !??!

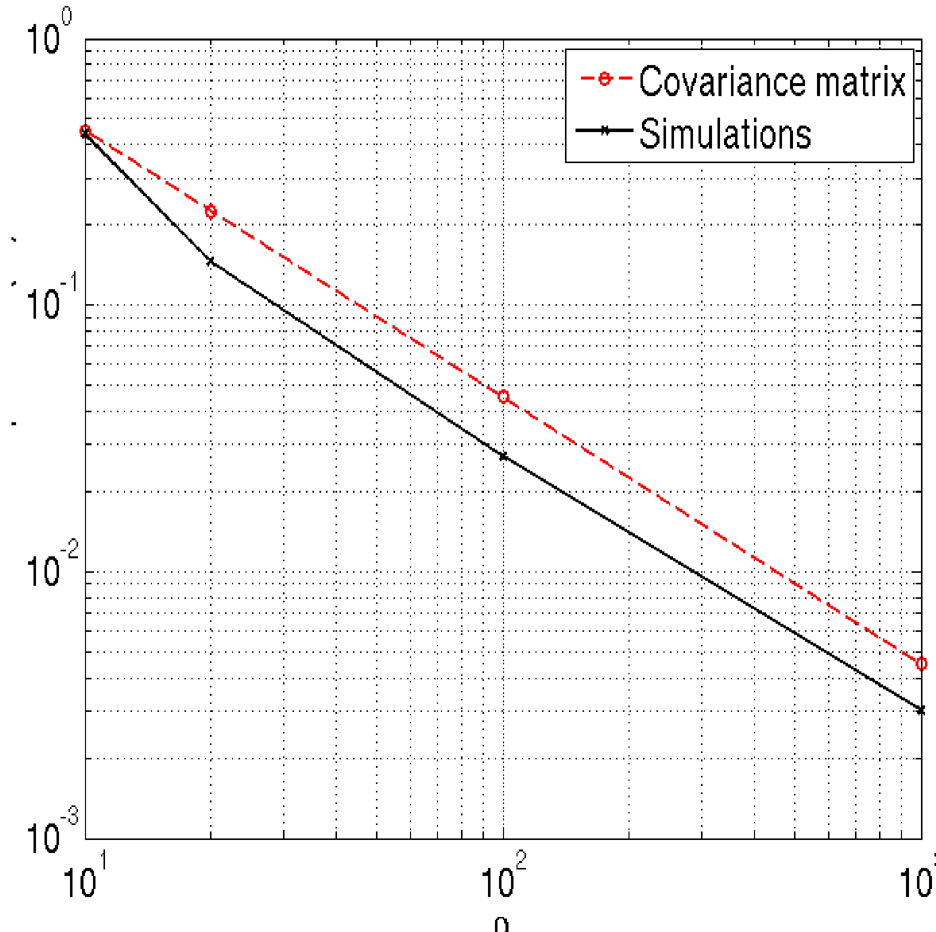




BBH simulations (5,5)

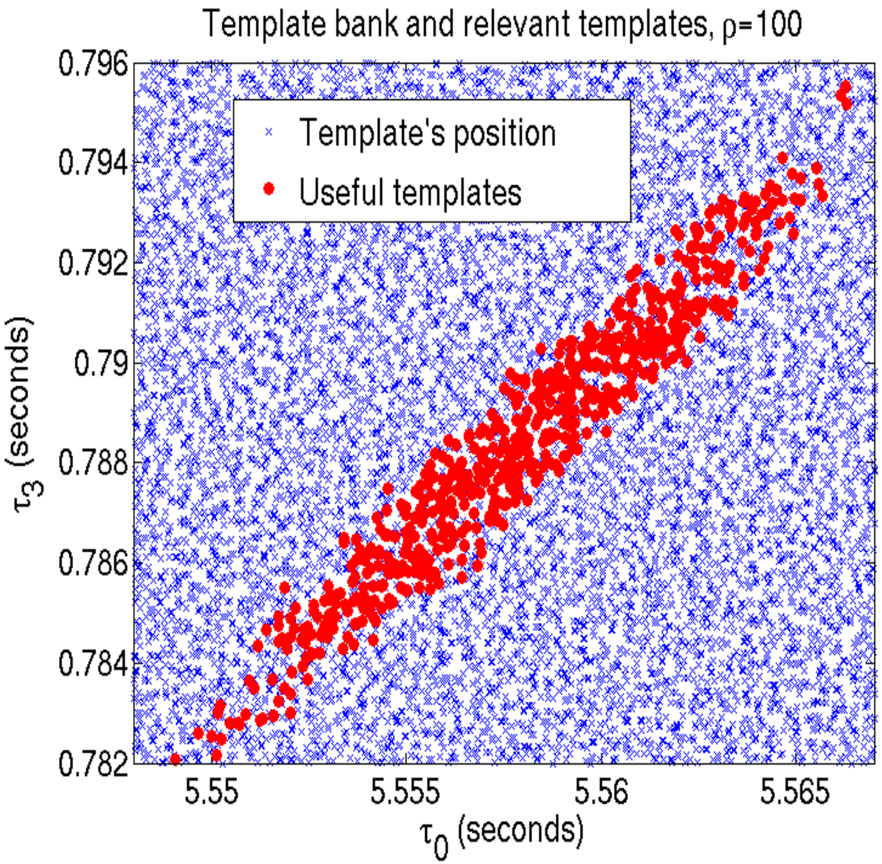
BBH (5,5) injections with phase at 3.5PN order. Simulations DO NOT agree with covariance results. We have a better accuracy with our simulations !??!

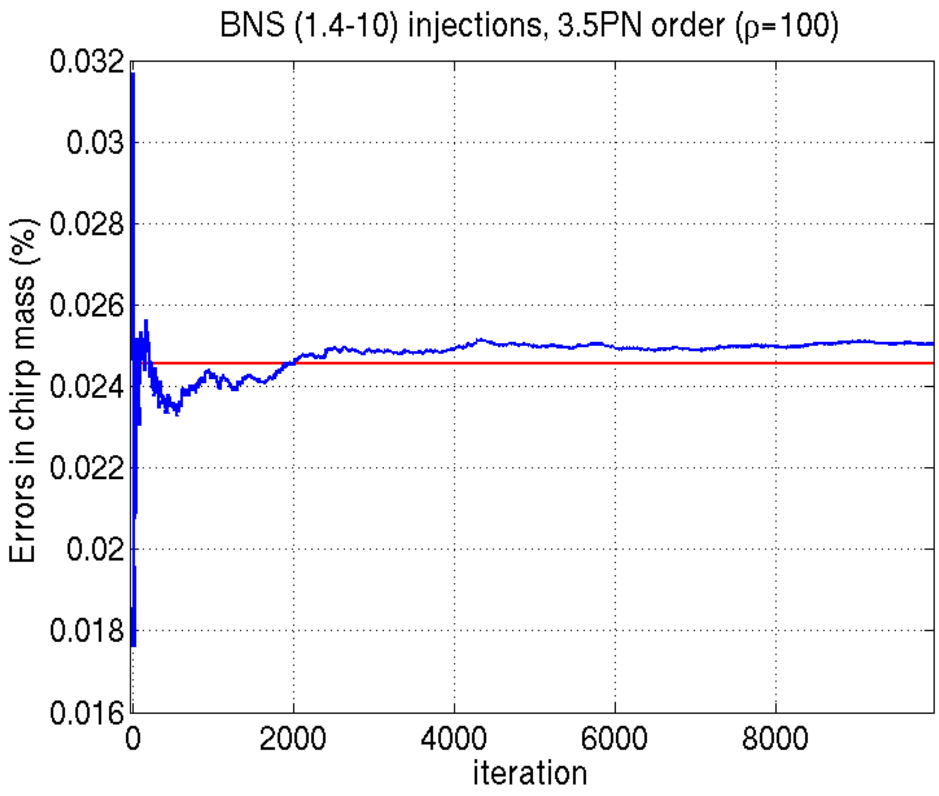




Sanity checks

Does the template bank span the noise fluctuations : yes (left picture)
Does the number of iterations sufficient : yes (right picture)



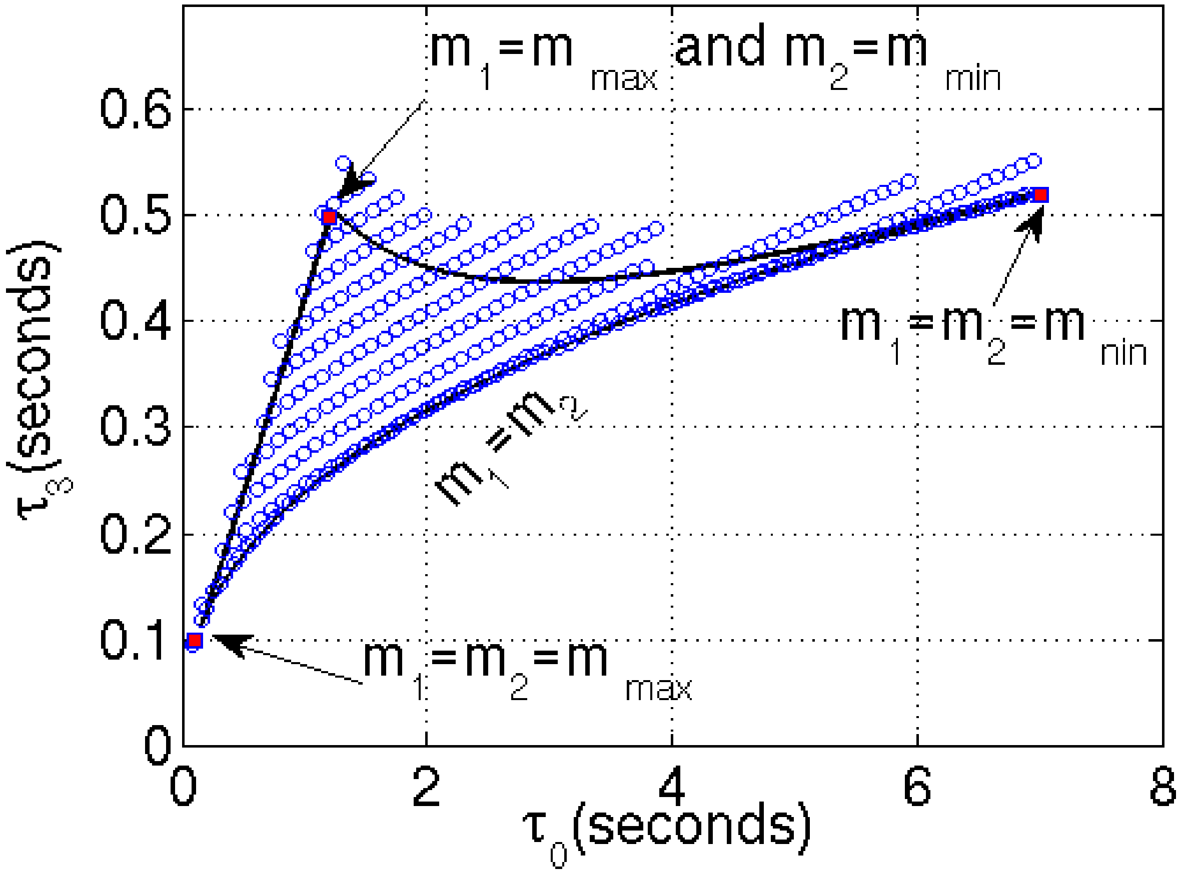


Solution to discrepancy between simulations and covariance matrix

Where is BHNS and where are BNS and BBH systems in the parameter space ?

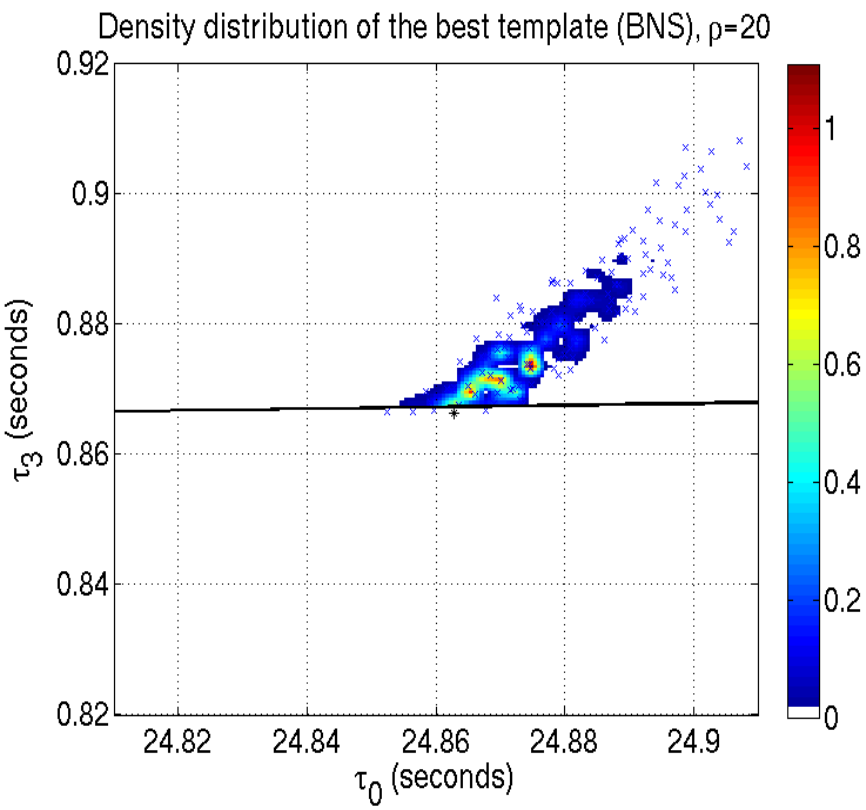
now let us remember that we are using a template bank with is very fine indeed, so this is Not the issue. However, we restrict the component masses so that eta <=0.25.

Let us use unphysical components masses to generate more templates. (M and eta remains physical)

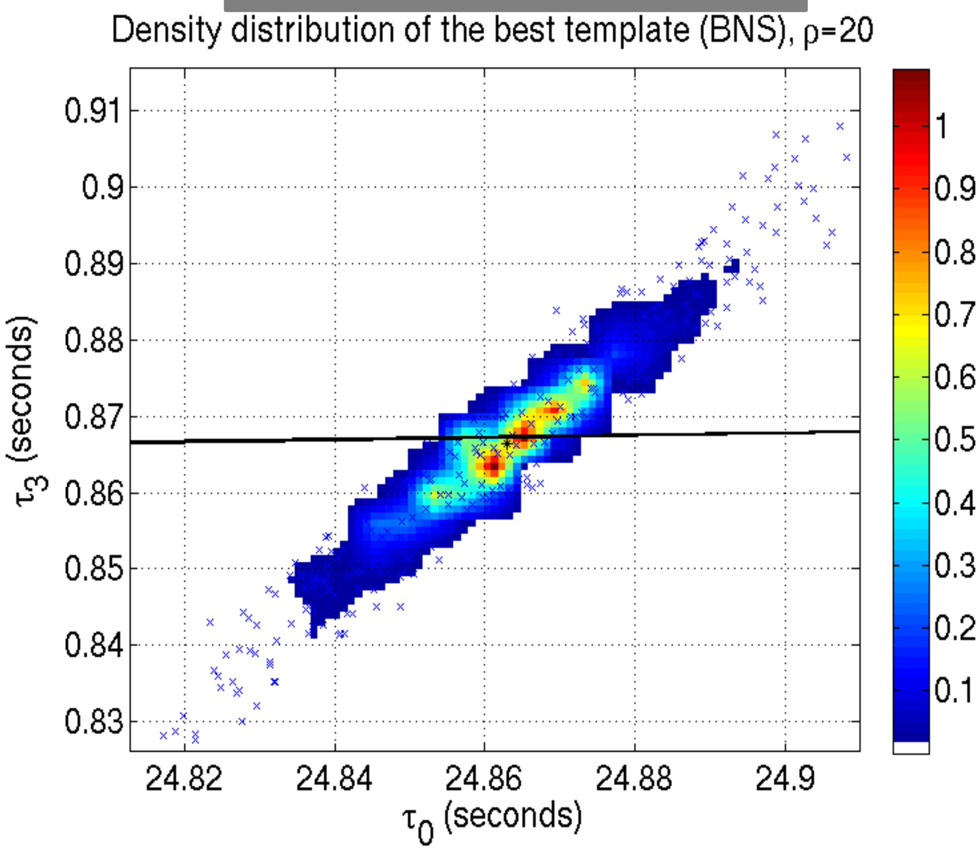


Solution to discrepancy between simulations and covariance matrix

Using physical masses only

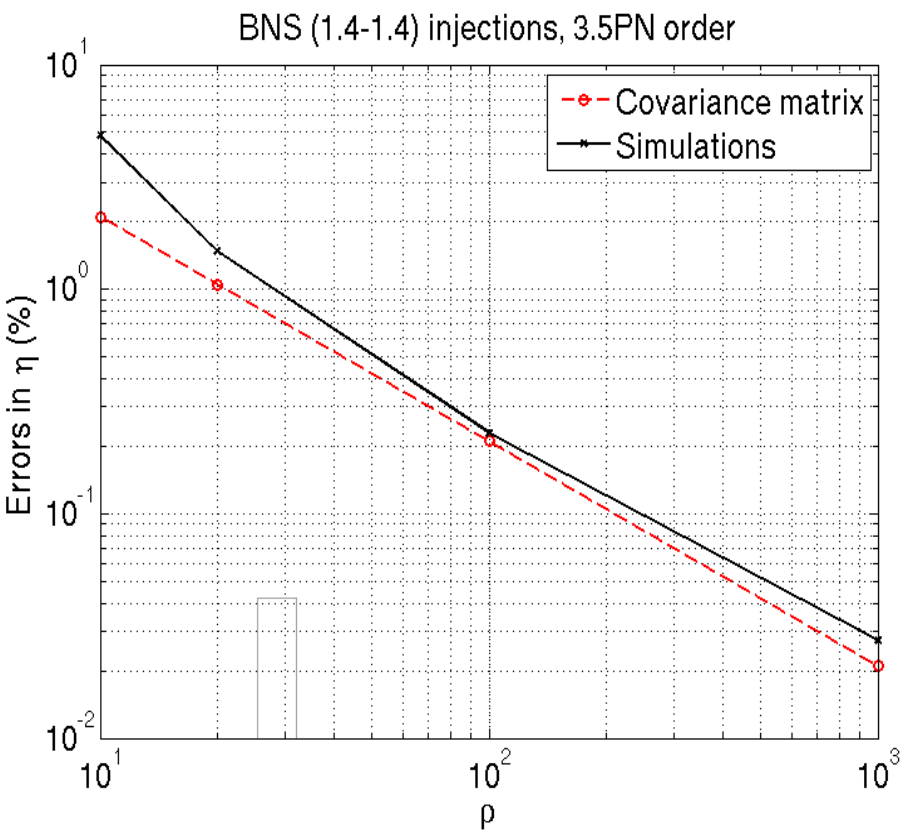


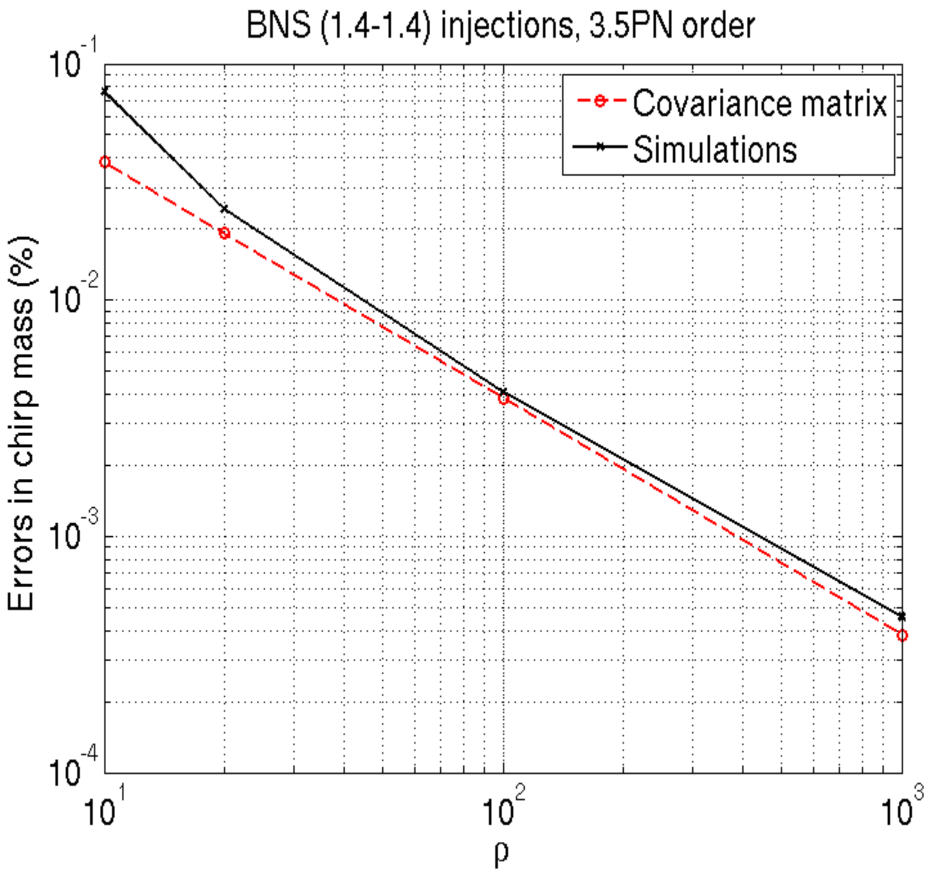
Using unphysical masses



BNS simulations, using unphysical mass templates

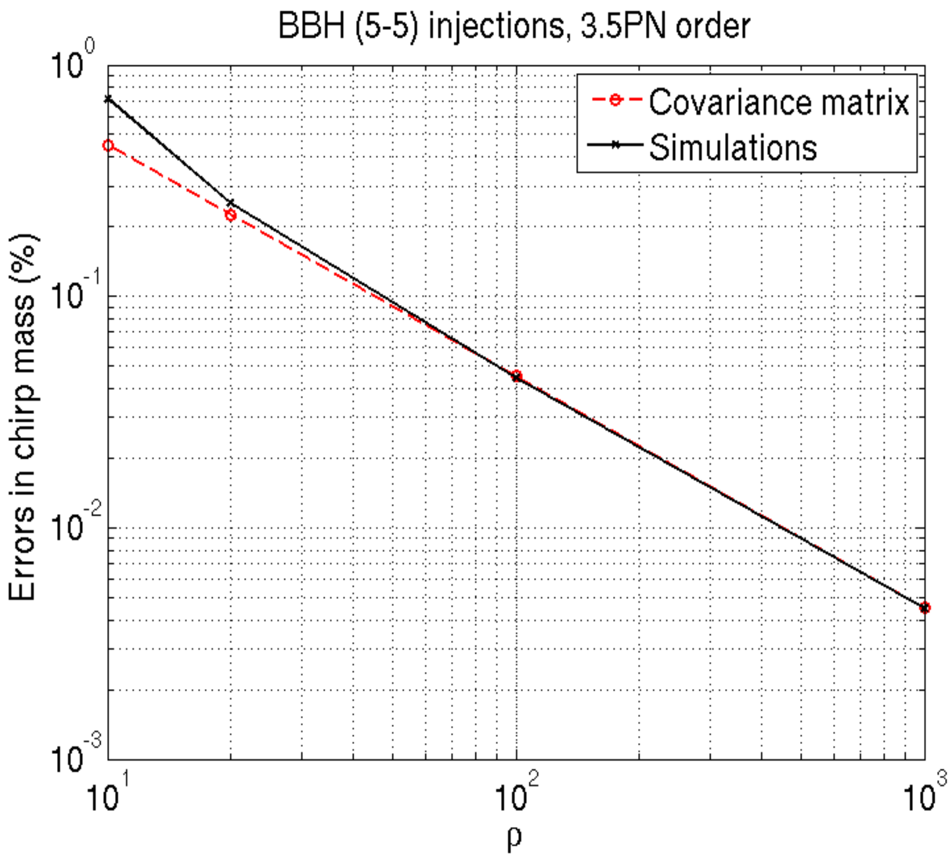
BNS (1.4,1.4) injections with phase at 3.5PN order. Simulations agrees with covariance results for SNR > 20 up to 1000 at least.

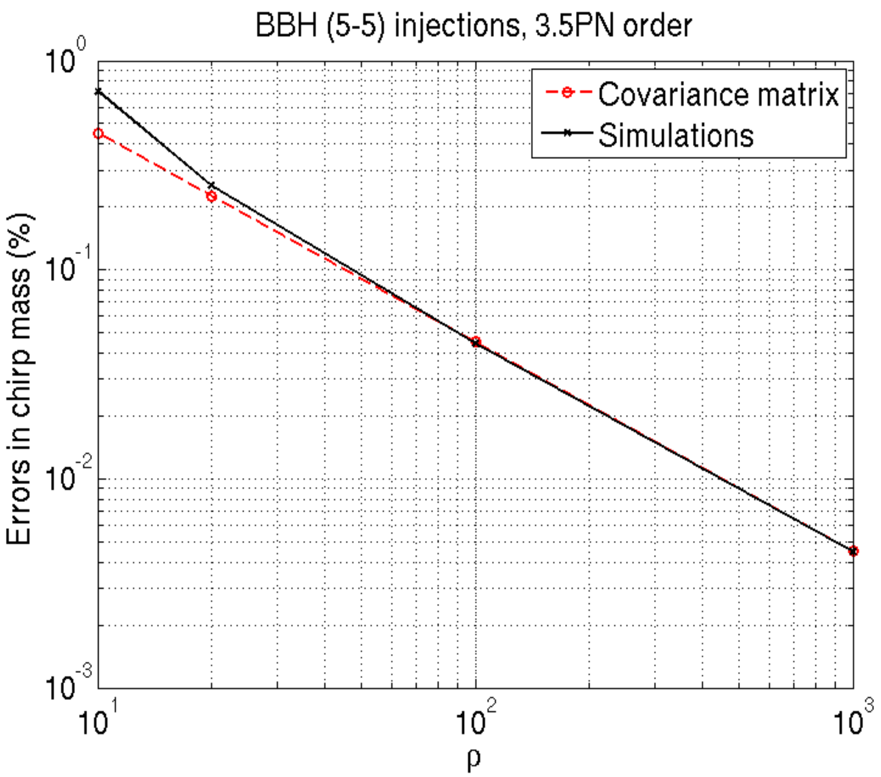




BBH (5,5) simulations, using unphysical mass templates

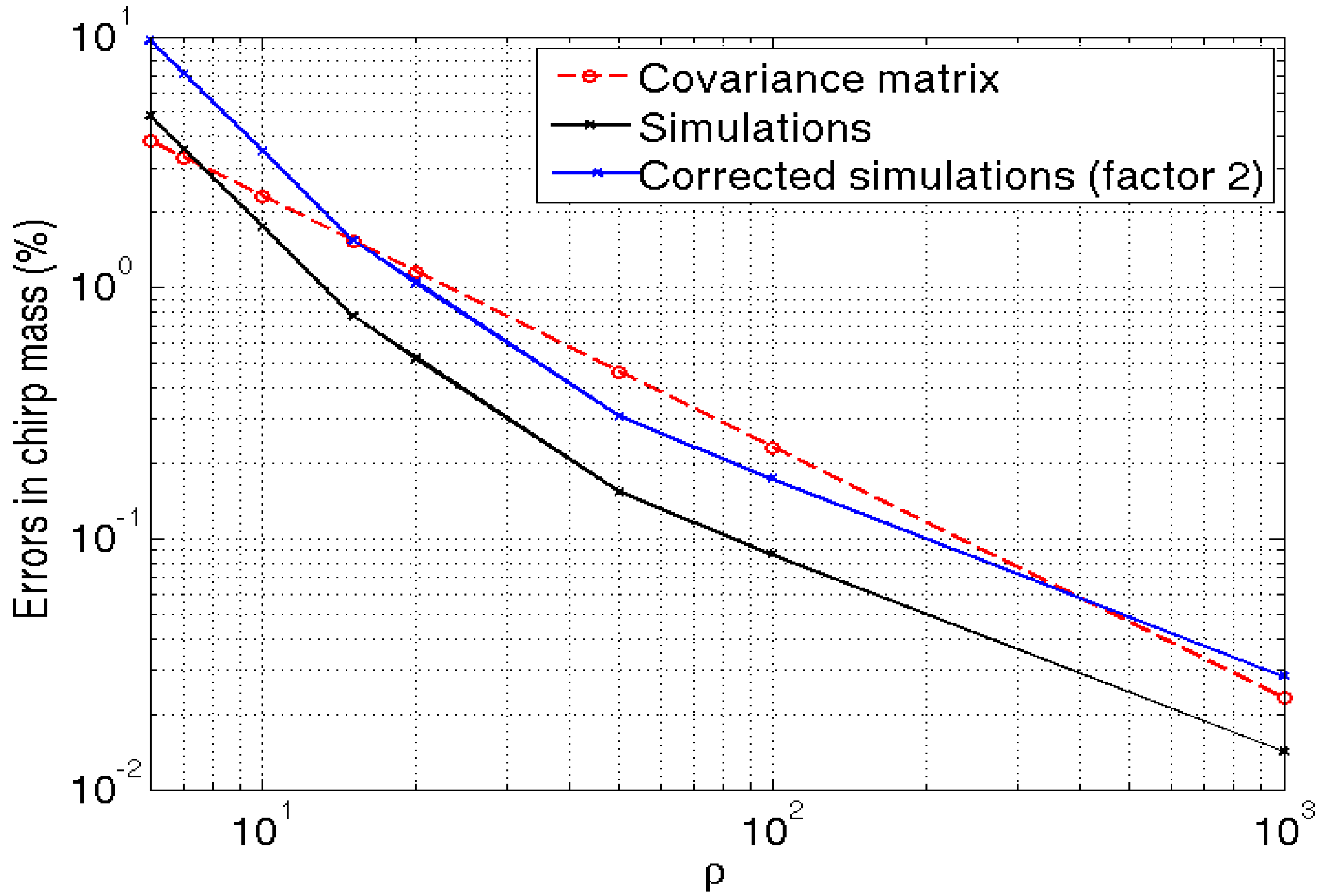
BBH (5,5) injections with phase at 3.5PN order. Simulations agrees with covariance results for SNR > 20 up to 1000 at least.





BBH (10,10) simulations

Even with unphysical mass templates high mass range leads to discrepancies



Conclusion and implications for ground based detectors

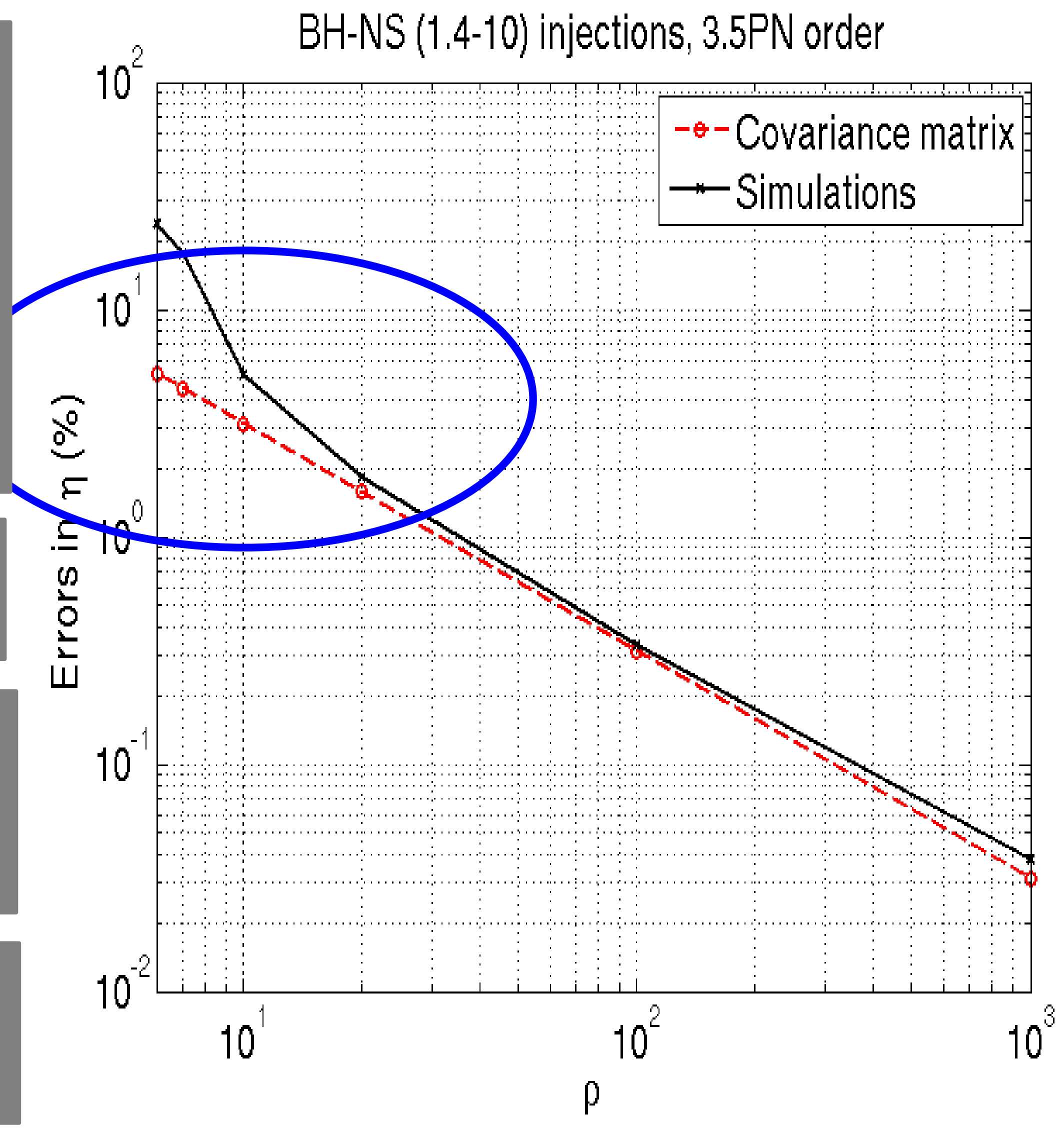
Our simulations shows good agreement with expectation for BHNS.

However, for symmetric systems, we need to use unphysical mass templates to get the same errors as covariance results. Otherwise, simulations give accuracy twice as better as expectation from covariance matrix!

At low SNR, we can estimate the actual errors using MC simulations

At SNR=6, there is a ratio of 4 with the theory. Next is to quantify precisely the impact of this discrepancy for LIGO (coincidence between detectors).

More simulations to investigate
1 – high mass range issues
2 – other approximant (EOB, Pade...)



The template bank

For detection, we use a standard template as used in LIGO analysis (T.Cokelaer, Phys. Review D 76, 10, 2007). With a minimal match of 97% (SNR loss of 3%), we need about 9200 templates to search for systems between 1 and 35 solar mass.

For parameter estimation, we need a finer placement. what is the minimal match required ? Right picture shows the span of a template (MM=97%) and distribution of parameter using a fine bank and injection with SNR=100. We need MM=99.9%. What about SNR=1000? 99.9999? Huge bank . Not feasible-> Hybrid adaptative bank.