Using the resampling technique to calculate the F-Statistic

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Time Domain Process

Frequency Domain Process

Raw Time Domain Data

Short Fourier Transforms

Heterodyne

Low Pass Filter + Downsample

Inverse Fourier Transform

Resample

Correct for Heterodyne in SSB Frame

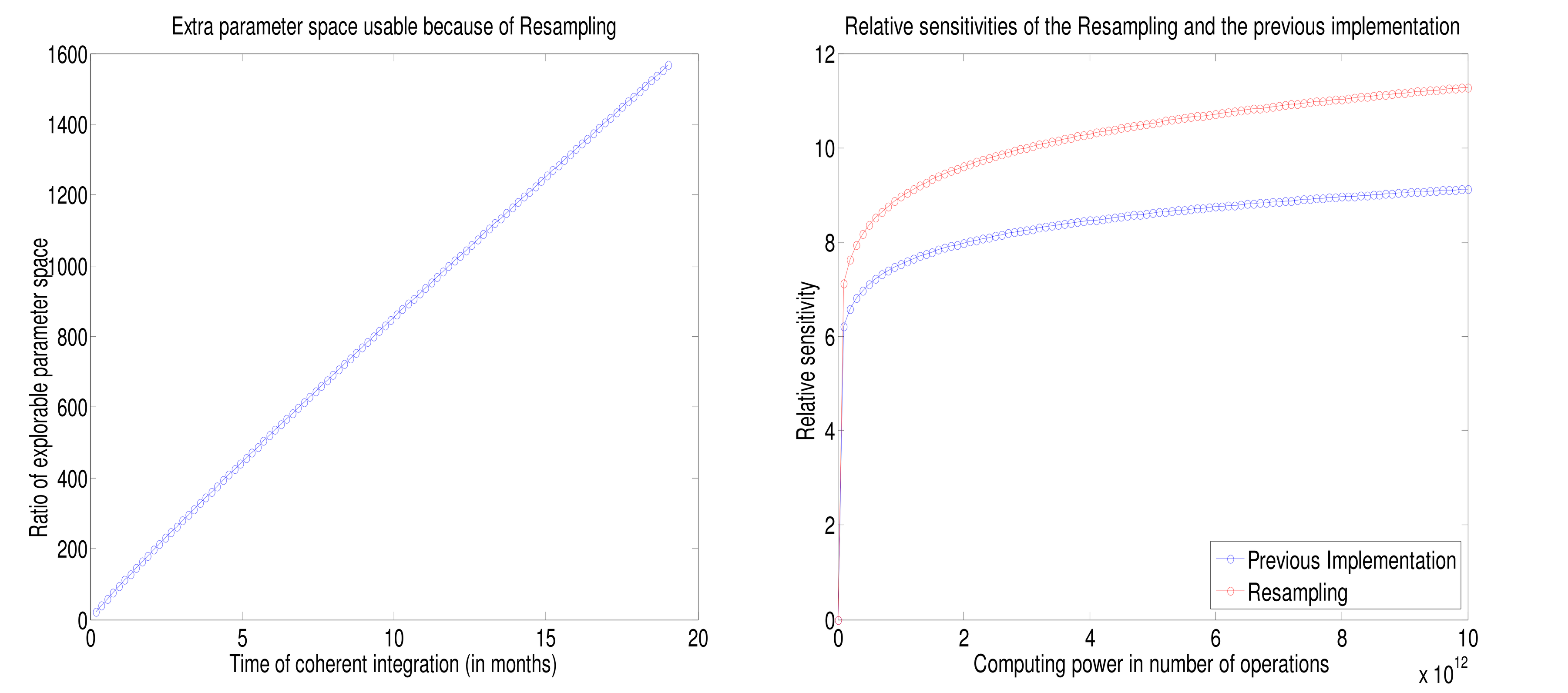
Correct for Frequency Spin-Down effect

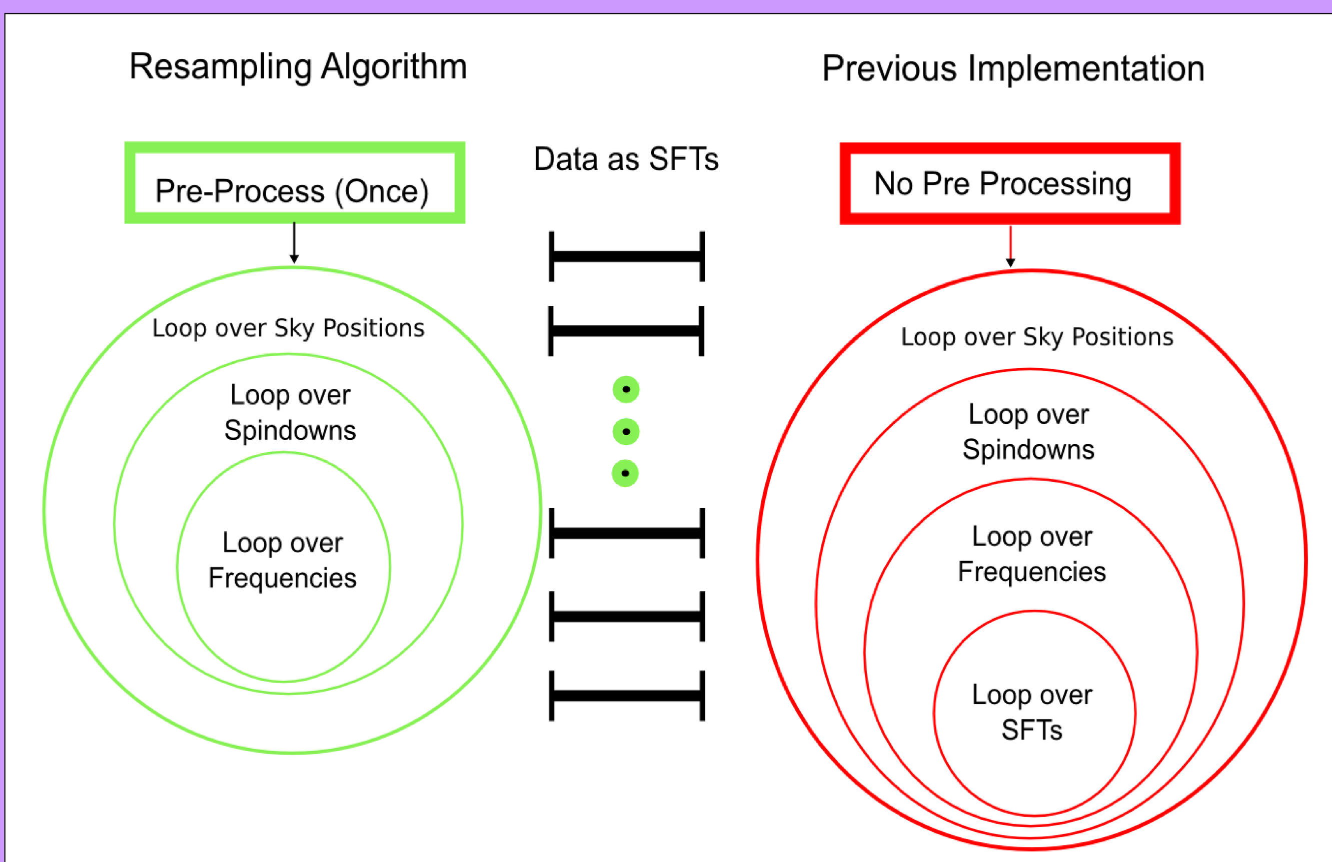
Calculate F_a and F_b using an FFT

The $\mathcal{F}$-Statistic

Abstract

An algorithm to calculate the F-statistic, a maximum likelihood statistic used to detect a source emitting continuous gravitational waves. We describe a practical way of dealing with computational issues by heterodyning and down-sampling the data. This algorithm is faster by a factor of the order of N/log(N), where N is the number of data-points used in the calculation. This will help continuous wave searches in LIGO in the exploration of previously computationally bound parameter spaces.





Why?

- Eliminate the Loop over Short Fourier Transforms (SFTs)
- Thus get a speed up of the order of the number of SFTs in your data set.
- The cost of the initial pre-processing is tiny as compared to the whole analysis

Challenges

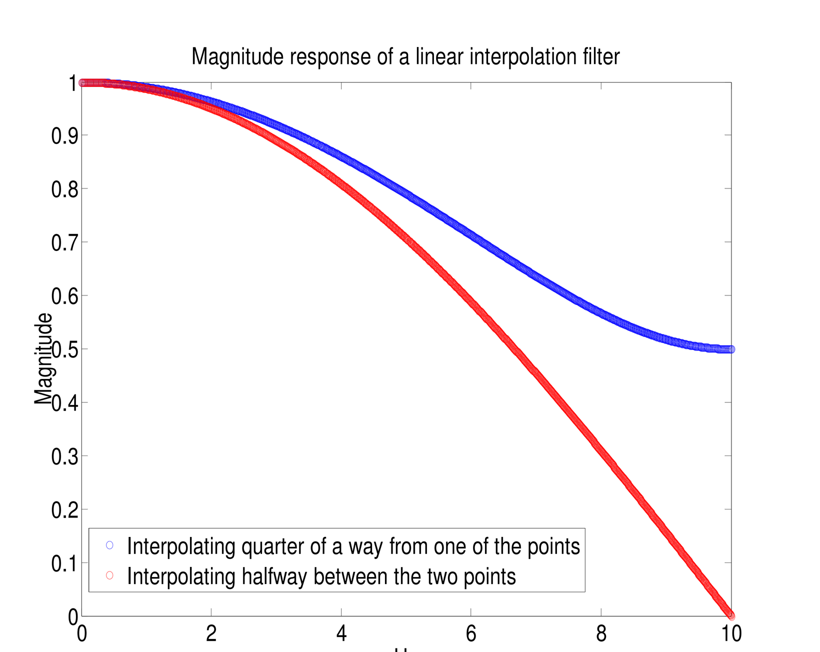
- Even a **modest search** over a few days time scale overwhelms current computing resources.
- Significant computing power exists in the form of **Einstein@Home**, which requires the data to be parcelled in small chunks, keeping the communication between servers and nodes to a minimum.
- An easy way to deal with potentially **huge parameter spaces** must be found. The scaling of the number of templates in an all sky search keeping only the first spin-down scales as the 5th power of T, where T is the time of observation.
- A **computational brick wall** is hit whenever an all sky fully coherent search is conducted

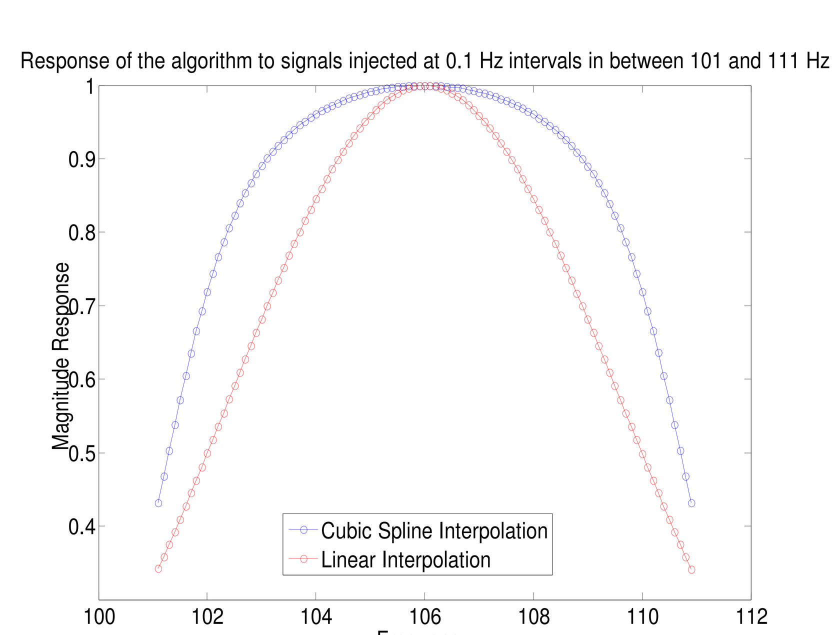
Subtle Issues

- **Heterodyning** is done in the detector's reference frame. A correction term needs to be used in the SSB frame.
- The LSC uses half-hour long SFTs and we need to use an interpolation method using the **Dirichlet Kernel** to create longer time baseline Fourier Transforms.
- The effects of **Interpolation** can be countered by pre-filtering the data with an inverse filter of the interpolator. In effect its like applying a high pass filter to the data before it is passed through the effective low pass filter.

Interpolation

- Interpolation acts like a low pass filter and reduces the power at higher frequencies in our band of analysis.





F-Statistic[1]

A maximum likelihood statistic called the F-statistic is used to search for gravitational waves from periodic sources. It is defined as -

$$\mathcal{F} = \frac{4}{S_h(f)T_0}\frac{B|F_a|^2 + A|F_b|^2 - 2CR(F_aF_b^*)}{D}$$

Where S_h is the power spectral density of the data and T_0 is the time of integration. A,B,C,D are functions of a(t) and b(t), the antenna patterns of the detector. And -

$$F_a(f) = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)a(t)e^{-2\pi i f(t+\Phi_m(t))}e^{-2\pi i\Phi_s(t)}dt$$

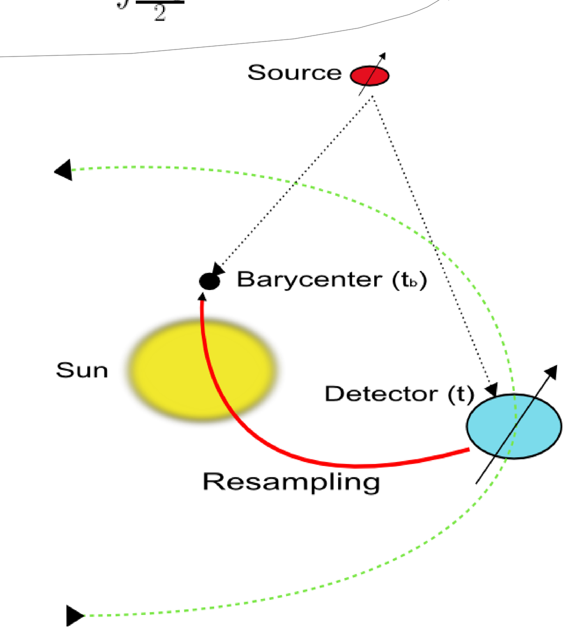
$$F_b(f) = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)b(t)e^{-2\pi i f(t+\Phi_m(t))}e^{-2\pi i\Phi_s(t)}dt$$

Where x(t) is the data, t is the time elapsed in the detector's frame of reference, and Φ_m is the difference in photon arrival times at the detector and the solar system barycenter (SSB) from a distant source. Φ_s is a phase factor, which determines the time lag due to the frequency spin down of the source.

Resampling[1]

$$F_a(f) = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t_b)a(t_b)e^{-2\pi i f(t_b)}e^{-2\pi i\Phi_s(t_b)}dt_b$$

$$F_b(f) = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t_b)b(t_b)e^{-2\pi i f(t_b)}e^{-2\pi i\Phi_s(t_b)}dt_b$$



References and Acknowledgments

[1] P.Jaranowski, A.Królak, B.Schutz, Physical Review D 58 , 063001, Data analysis of gravitational wave signals from spinning neutron stars.
[2] Ligo Analysis Library , LAL
[3] GNU Scientific Library , GSL.
[4] Fastest Fourier Transform in the West (FFTW3)

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