Extracting Extreme Mass Ratio Inspiral Parameters via Time-Frequency Methods

Ilya Mandel, Jonathan Gair, Linqing Wen
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Analysis techniques

- Template-bank-based matched filtering
- Markov-Chain Monte Carlo
- Time-frequency analysis:
  - Build a spectrogram, find tracks, estimate parameters
  - "Cheap and dirty"
  - Could be used for detections and for initial parameter estimates before further refinements

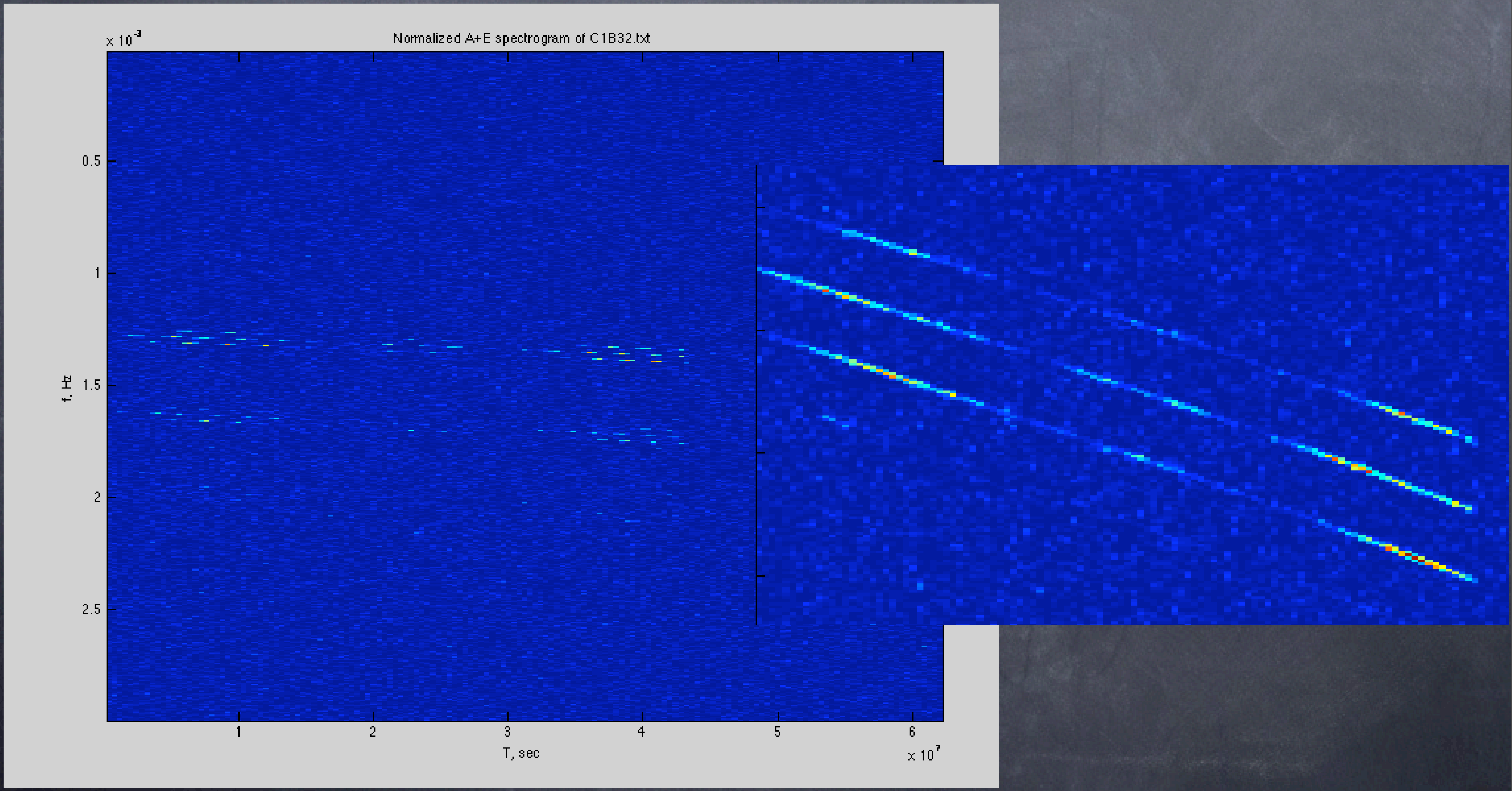
Time-frequency analysis

1. Construct a spectrogram
2. Find tracks in the spectrogram
3. Roughly estimate parameters from track shape
4. Refine parameter estimates using full information on track shape and power along tracks

Building a spectrogram

- Form orthogonal A and E channels
- Pick a time bin length
- For each time bin, FFT the (Hanning windowed) data, compute power in each time-frequency pixel
- Normalize the power by dividing by noise power spectral density
- Add spectrograms of A + E channels

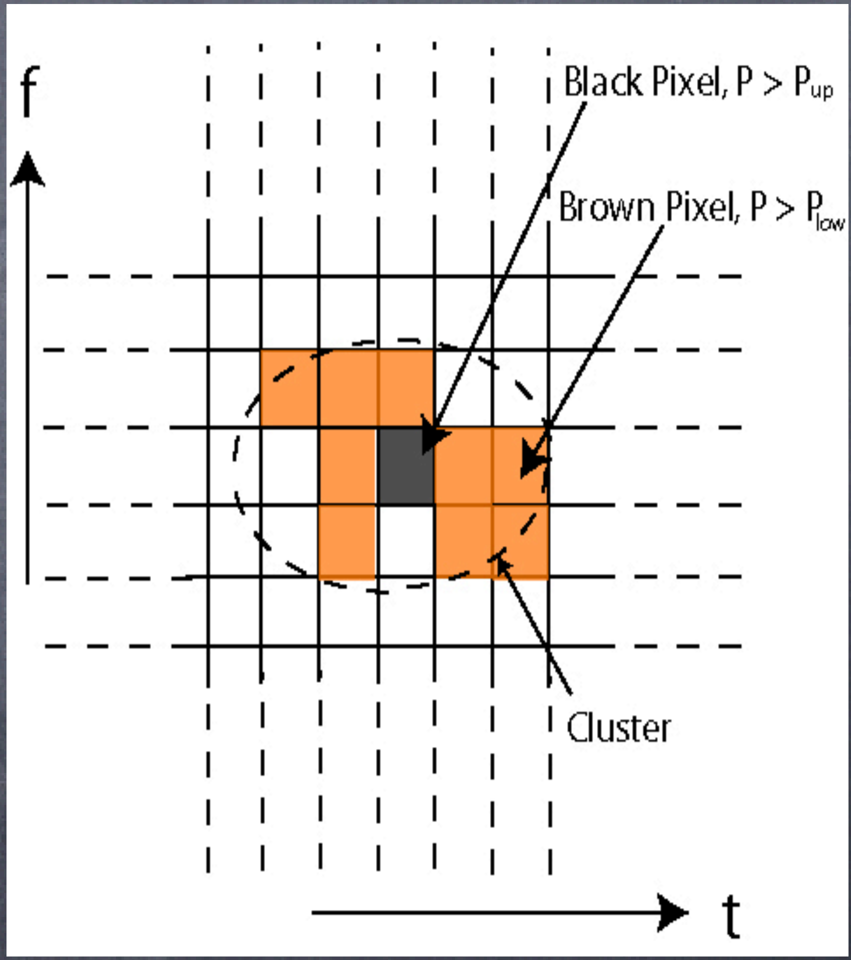
Typical spectrogram: C1B.3.2



Track search - HACR

Hierarchical Algorithm for Clusters and Ridges [Gair & Jones, 2006]

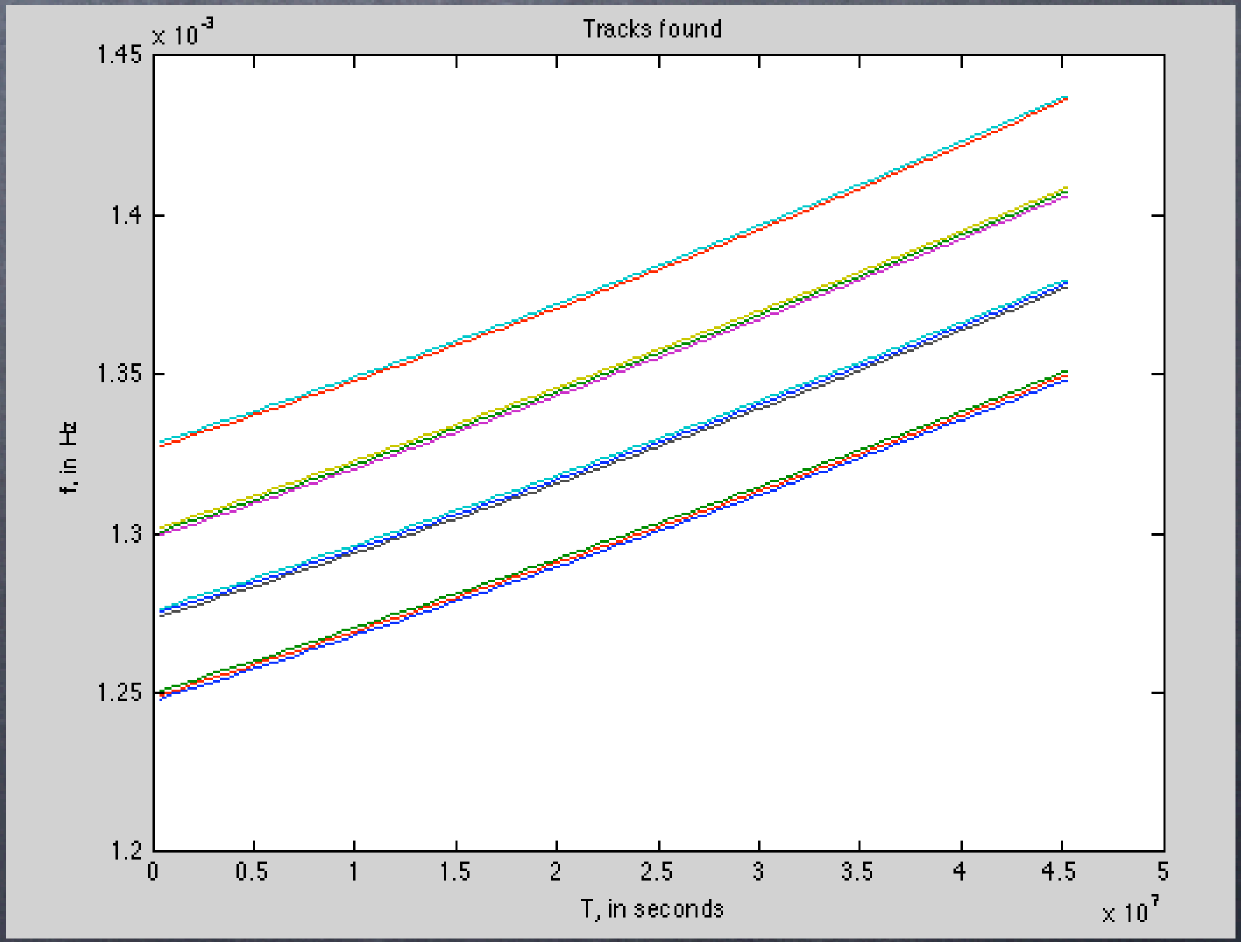
1. Search for 'black pixels', i.e., pixels with power greater than an upper threshold, P_{up}.
2. Count the number of 'brown pixels' (i.e., pixels with power greater than a lower threshold, P_{low}) in a contiguous group (a cluster) connected to each black pixel.
3. If a cluster has more than a threshold number of pixels, N_p, it constitutes a detection.



Problem: HACR can't find tracks that are visible by eye. That's because we know what to look for by eye!

Track search - CATS

- Chirp-based Algorithm for Track Search
- An EMRI track will be a chirp, characterized by the frequency and its first two time derivatives
- Find the track in the (f, df/dt, d^2f/dt^2) space with the most summed power over track
- Remove brightest track & repeat to find sidebands



CATS: Sanity Checks

- Compare detected tracks with visual observation of spectrogram
- Check if tracks have sidetracks one pixel off
- Check for typical EMRI structure with sidebands due to orbital-plane precession
- Check if track parameters are reasonable (e.g., f, df/dt for late track sections should be greater than for early track sections)
- Future thresholds will be set by choosing FAP

Rough parameter estimates

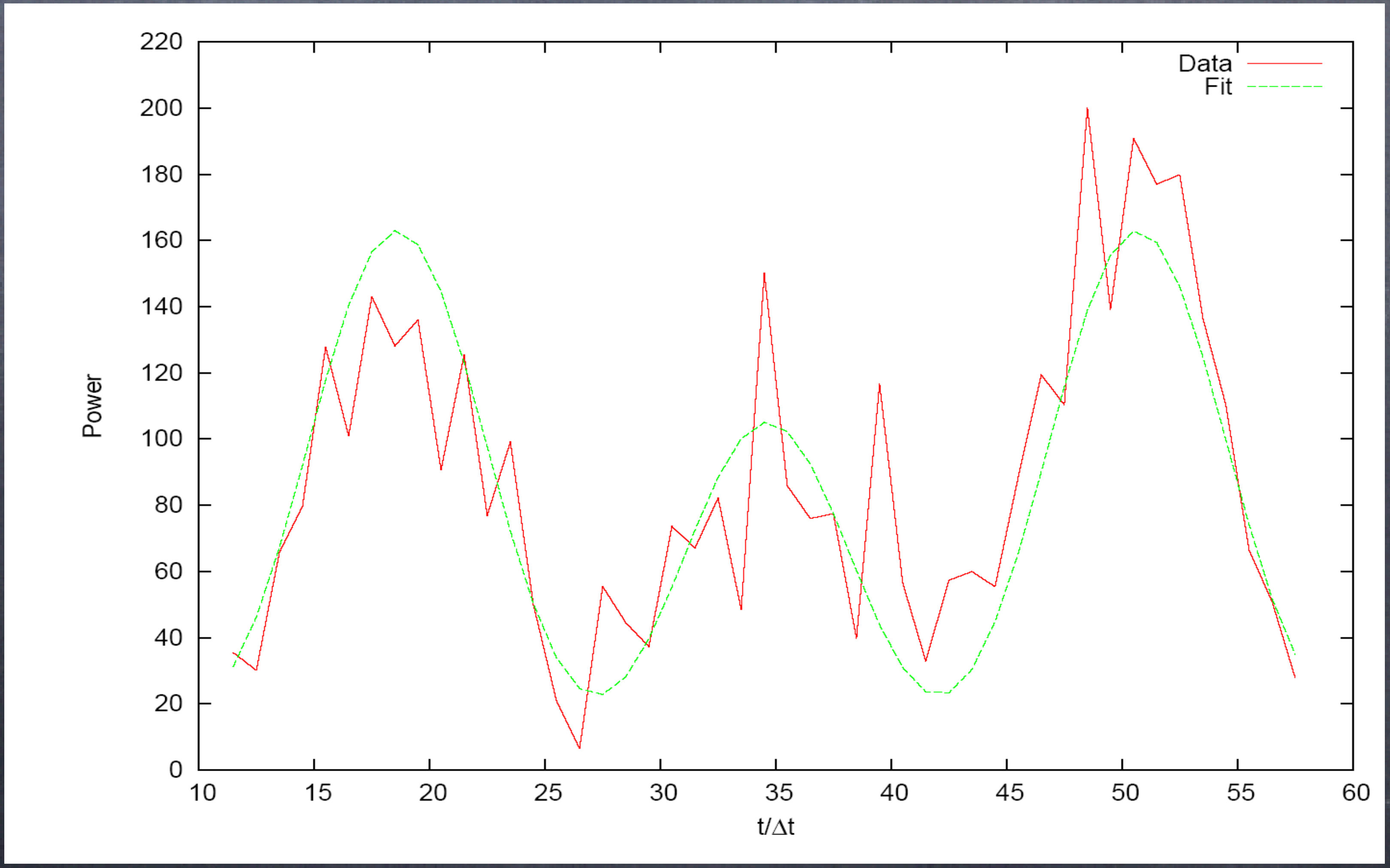
- Harmonic separation yields radial frequency
- Together with absolute frequency, this yields periapsis precession frequency
- Sideband separation yields orbital-plane precession frequency
- With plunge time and frequency evolution, these determine the intrinsic parameters: masses, spin, eccentricity, spin-orbit angle
- Yearly variation in power along track yields sky position

Sky position estimation

- Estimate sky position by fitting the signal amplitude variation along the strongest harmonic to an approximate form [Cutler 1998] (valid in low-frequency limit):

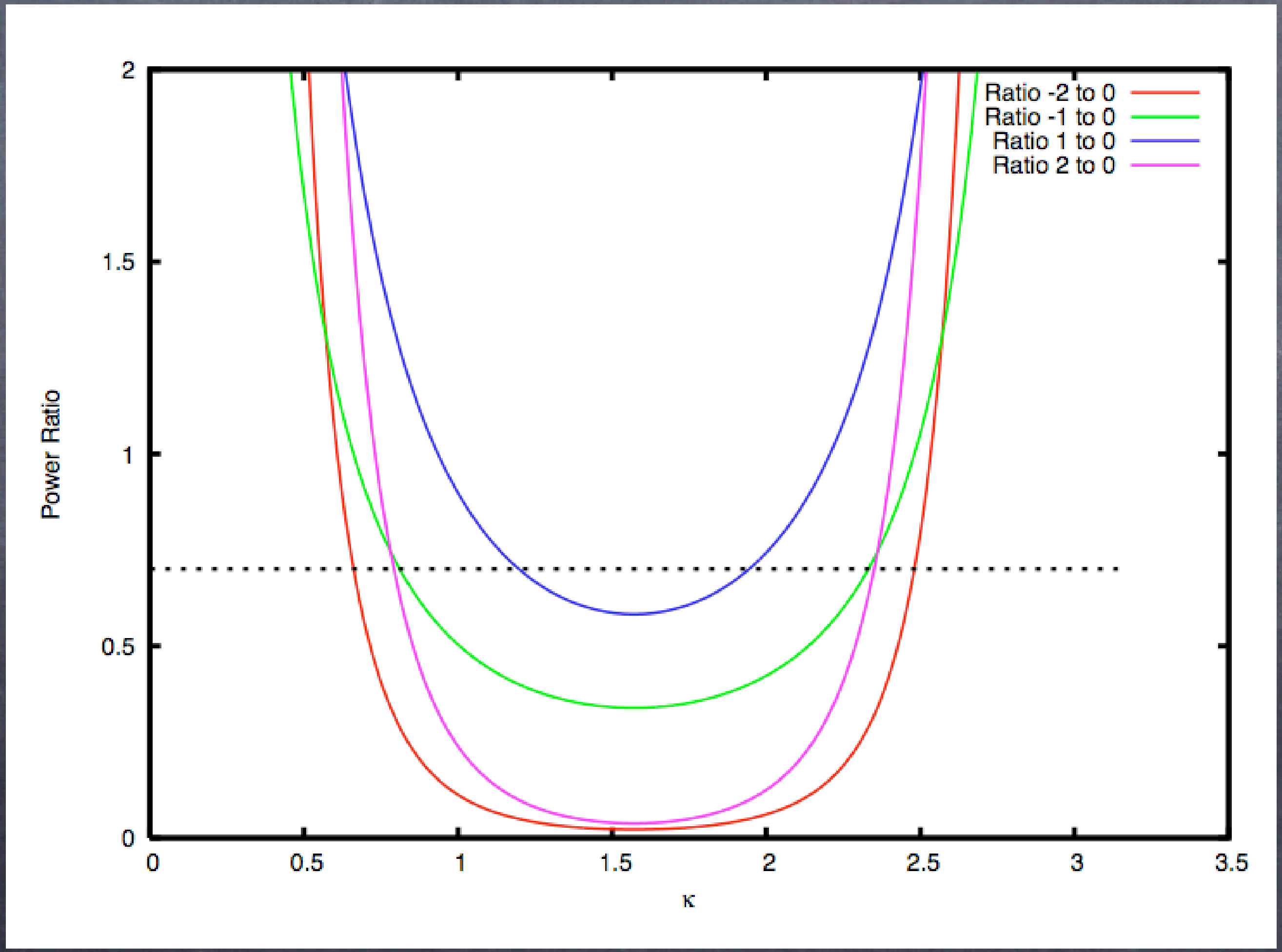
$$\mathcal{M}(t) \quad \propto (F_I^+)^2 + (F_I^\times)^2 + (F_{II}^+)^2 + (F_{II}^\times)^2 \propto 1 + 6\cos^2\theta + \cos^4\theta$$

$$\text{where} \quad \cos\theta = \frac{1}{2}\cos\theta_S - \frac{\sqrt{3}}{2}\sin\theta_S \cos\left(\frac{2\pi t}{T} - \phi_S\right)$$



Inclination estimation

- Relative power in different sidebands of a given harmonic depends only on the spin-orbit angle and the inclination of SMBH spin to line of sight



True inclination: 1.295; submitted: 1.262

Parameter refinement

- Metropolis-Hastings Monte Carlo, runs on lists of spectrogram pixels along chirp tracks
- Uses full information (e.g., relative power in harmonics is a function of eccentricity, relative power in sidebands depends on angle between SMBH spin and line of sight...)
- Seeded with rough parameter estimates, single 100,000 point chain
- For now, search for most likely set of parameters - don't estimate PDF, FAP, etc.

Results

Challenge	Eclip. Lat.	Eclip. Long.	Spin	CO mass/$M_\odot$	SMBH mass/$M_\odot$	ν_{init} (Hz)	e_{init}	λ
C1B.3.1 Rough	0.491	4.9	0.698	10.5	9.56×10^6		0.208	0.424
C1B.3.1 Final	0.4941	4.939	0.6667	10.404	9.787×10^6	0.1921	0.1886	0.1938
C1B.3.1 True	0.5526	4.9104	0.6982	10.2961	9.5180×10^6	0.1920	0.2144	0.4395
C1B.3.2 Rough	0.393	4.59	0.641	9.593	5.118×10^6		0.214	1.48
C1B.3.2 Final	0.4028	4.656	0.6371	9.817	5.250×10^6	0.3423	0.2017	1.423
C1B.3.2 True	0.3597	4.6826	0.6380	9.7711	5.2156×10^6	0.3423	0.2079	1.4358
C1B.3.3 Rough	1.100	0.628	0.525	9.713	5.2285×10^6		0.185	0.921
C1B.3.3 Final	1.013	0.7348	0.5318	9.756	5.250×10^6	0.3426	0.1936	0.9091
C1B.3.3 True	0.9817	0.7097	0.5333	9.6973	5.2197×10^6	0.3426	0.1993	0.9282
C1B.3.4 Rough	1.508	1.19	0.62	10.04	9.51×10^5		0.249	1.663
C1B.3.4 Final	0.7805	4.168	0.6133	9.989	9.518×10^5	0.8512	0.4564	1.658
C1B.3.4 True	-0.9802	0.9787	0.6251	10.1047	9.558×10^5	0.8514	0.4506	1.6707
C1B.3.5 Rough	1.493	0.565	0.610	9.925	9.504×10^5		0.250	1.39
C1B.3.5 Final	0.726	4.45	0.618	10.49	9.500×10^5	0.887	0.417	1.47
C1B.3.5 True	-1.1092	1.0876	0.6583	9.7905	1.0334×10^6	0.8322	0.4269	2.3196

Extra slides

Results

Parameter	True value	EtfAG	BBGP	MT (2yr)
Ecliptic Latitude (radian)	0.5526	0.4941	0.6456	0.5342
Ecliptic Longitude (radian)	4.9104	4.939	4.9473	4.8935
Spin (S_1/M^2)	0.6982	0.6667	0.6362	0.6986
Compact object mass $/M_\odot$	10.2961	10.404	10.500	10.2897
SMBH mass $/(10^6 M_\odot)$	9.5180	9.787	10.3586	9.5042
Initial Azimuthal Frequency (mHz)	0.1920	0.1921	0.1871	0.1920
Initial Eccentricity	0.2144	0.1886	0.1581	0.2157
Lambda Angle (radian)	0.4395	0.1938	0.5080	0.4458

TABLE I: True values and estimates of Challenge 1B.3.1 parameters.

Parameter	True value	EtfAG	BBGP	MT (2yr)
Ecliptic Latitude (radian)	0.3597	0.4028	0.8743	0.3470
Ecliptic Longitude (radian)	4.6826	4.656	1.1145	4.7365
Spin (S_1/M^2)	0.6380	0.6371 (0.6359)	0.6397	0.6194
Compact object mass $/M_\odot$	9.7711	9.817 (9.697)	9.7751	9.8353
SMBH mass $/(10^6 M_\odot)$	5.2156	5.250 (5.180)	5.2076	5.0407
Initial Azimuthal Frequency (mHz)	0.3423	0.3423 (0.3421)	0.3422	0.3543
Initial Eccentricity	0.2079	0.2017 (0.2145)	0.2094	0.1977
Lambda Angle (radian)	1.4358	1.423 (1.458)	1.4399	1.1474

TABLE II: True values and estimates of Challenge 1B.3.2 parameters.

Parameter	True value	EtfAG	BBGP	MT (2yr)
Ecliptic Latitude (radian)	0.9817	1.013	0.6948	0.8409
Ecliptic Longitude (radian)	0.7097	0.7348	3.8708	0.8319
Spin (S_1/M^2)	0.5333	0.5318	0.5965	0.5465
Compact object mass $/M_\odot$	9.6973	9.756	10.1932	9.5665
SMBH mass $/(10^6 M_\odot)$	5.2197	5.250	5.2344	4.9231
Initial Azimuthal Frequency (mHz)	0.3426	0.3426	0.3424	0.3425
Initial Eccentricity	0.1993	0.1936	0.1965	0.2496
Lambda Angle (radian)	0.9282	0.9091	0.7588	0.8958

TABLE III: True values and estimates of Challenge 1B.3.3 parameters.

Parameter	True value	EtfAG	BBGP	MT (2yr)
Ecliptic Latitude (radian)	-0.9802	0.7805	0.8134	
Ecliptic Longitude (radian)	0.9787	4.168	3.2806	
Spin (S_1/M^2)	0.6251	0.6133	0.6301	
Compact object mass $/M_\odot$	10.1047	9.989	10.0849	
SMBH mass $/(10^6 M_\odot)$	0.9558	0.9518	1.0439	
Initial Azimuthal Frequency (mHz)	0.8514	0.8512	0.7951	
Initial Eccentricity	0.4506	0.4564	0.4408	
Lambda Angle (radian)	1.6707	1.658	1.7804	

TABLE IV: True values and estimates of Challenge 1B.3.4 parameters.

Parameter	True value	EtfAG	BBGP	MT (2yr)
Ecliptic Latitude (radian)	-1.1092	0.726	0.3905	
Ecliptic Longitude (radian)	1.0876	4.45	1.9690	
Spin (S_1/M^2)	0.6583	0.618	0.6770	
Compact object mass $/M_\odot$	9.7905	10.49	9.8849	
SMBH mass $/(10^6 M_\odot)$	1.0334	0.9500	0.9787	
Initial Azimuthal Frequency (mHz)	0.8322	0.887	0.8339	
Initial Eccentricity	0.4269	0.417	0.4295	
Lambda Angle (radian)	2.3196	1.47	2.5092	

TABLE V: True values and estimates of Challenge 1B.3.5 parameters.